Radiation transport in high pressure Hg/Xe discharges

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Monte Carlo simulation and a ray-tracing method are applied to calculate the radiation transport in Hg-Xe high-pressure discharges with cylindrical symmetry. The spectral intensity along a line of sight is calculated by solving the equation of radiative transfer. The radiation flux through the tube surface is found by means of integration of the spectral intensity over all solid angles. The Monte Carlo simulation delivers in a direct manner the number of photons escaping from the discharge corresponding to the radiative flux. In both approaches a temperature distribution derived from experimental observations is used. The spectral lineshape is described in terms of Van der Waals and Stark broadening, and the quasi-static approximation.

1. Introduction

The transport of radiation is an important issue for overall plasma characteristics. Especially in light sources, a description of the power balance in detail is needed in order to be able to study the influence of the plasma properties on light characteristics. In high pressure mercury discharges, the radiation extends over a wide spectral range with different characteristics. There are spectral lines for which the arc is "medium-thick". Therefore the radiation influences all parts of the plasma. The local net emission depends on the temperature and density distribution in the whole plasma column.

In this work, two approaches are applied to study the radiation transport in high pressure Hg/Xe discharges-Monte Carlo simulation and solving the equation of radiative transfer along probe lines with a consequent integration over all solid angles.

2. General conditions

In the present study, a cylindrically symmetric high pressure arc in Hg/Xe is considered. More details concerning the geometry and discharge parameters are given in [1]. Assuming LTE conditions and constant pressure in the discharge, the electron and neutral atom densities may be obtained from the equation of state of ideal gases and the Saha equation, and the temperature is obtained from experiments [1]. The relative densities of the emitters and absorbers are calculated from the temperature distribution and the Saha-Boltzmann LTE relations. The LTE expressions for the emission and absorption coefficients (stimulated emission is neglected) are used with a normalized line profile obtained as a convolution of Lorentz and Van der Waals quasi-static profiles [2]. The isotope displacement and hyperfine structure of the mercury atoms is taken into account [3]. The various states of odd isotopes are treated as independent species.

3. Monte Carlo simulation

The Monte Carlo simulations follow the algorithm similar to that used in [4]. Photons are launched from a specified distribution of emitters with a frequency chosen at random from the overall lineshape. The selection of the initial isotope is made with weighting according to the relative concentrations of each species. The direction of emission is selected from the three-dimensional isotropic distribution. The length l of the free flight of the photon before being reabsorbed is sampled according to

$$l = -\frac{\ln w}{\sum_i \kappa_i(\nu)}, \qquad (1)$$

where $\kappa_i(\nu)$ is the absorption coefficient of the isotope i, and w is a uniform random number in the interval (0,1). Then the position of reabsorption is calculated. If it lies within the computational volume in radial direction, the position is recorded, and the absorbing species is chosen with a probability proportional to its weight in the sum of absorption coefficients. If the point of reabsorption lies outside, the photon is recorded as escaped, and a new one is started. The procedure is repeated until a given number of photons (statistical precision) is reached.

4. Ray-tracing method

The method is based on the use of a number of probe lines to sample the radiation coming from different directions. The equation of radiative transfer [5] is solved along the probe lines:

$$\frac{dI_\nu}{ds} = \varepsilon_\nu - \kappa I_\nu, \qquad (2)$$

where I_ν ($Wm^{-2}Hz^{-1}sr^{-1}$) is the radiation intensity along a path s, and ε_ν ($Wm^{-3}Hz^{-1}sr^{-1}$) is the local emission coefficient. Since a cylindrical geometry is considered all quantities that determine absorption and emission are a function of the radius only. Then a probe line 1-1 is equivalent to a line 1a-1a (Fig. 1). The intensities I_ν calculated for lines of sight at different distances d from the arc axis (various path lengths s) correspond to the measured ones in side-on spectroscopic experiments. This allows one a direct comparison between calculations and experimental data.

The radiative flux F_ν (WHz^{-1}) across a surface A can be computed from I_ν (Fig. 1):

$$dF_\nu = I_\nu\left(\vec{A}\cdot d\vec{\Omega}\right) = I_\nu A \sin^2\theta \cos\phi d\theta d\phi \qquad (3)$$

The final result obtained after integrating Eq.(3) corresponds directly to the Monte Carlo simulations.

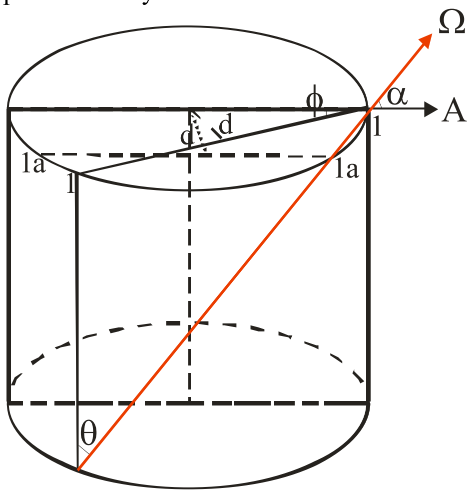

Fig. 1 Ray-tracing in direction Ω and in the circular cross section of the discharge tube.

4. Results

As a case study Monte Carlo simulations and ray-tracing calculations are performed for the Hg 546.074 nm line emitted from the LTE high pressure discharge P1 in Hg/Xe [1]. The temperature distribution used for the calculations is derived from experimental observations. The broadening constants for Stark and Van der Waals broadening are adjusted in such a way that a good agreement between calculated and observed line profiles for a line of sight intersecting the centre of the discharge can be achieved. These values for mercury perturbing atoms are C_6=0.25x10^{-42}m^6/s and $C_{6,qs}$=0.5x10^{-42}m^6/s, and C_4=0.45x10^{-22}m^4/s for Stark broadening. The constant for Van der Waals broadening with Xe atoms as perturbers is taken C_6=1.56x10^{-41}m^6/s as estimated from the polarizability. It was not adjusted because of the minor presence of Xe in the discharge considered here.

Fig. 2 presents the results from the ray-tracing calculations. The spectral intensity I_ν is given for

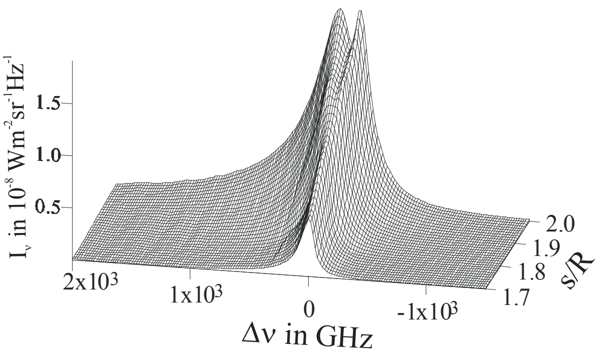

Fig. 2 Calculated line profiles of Hg 546.074nm for various path length *s*.

different path lengths at $\theta=\pi/2$. The self-reversal appears near the discharge axis. The contribution of the outer part of the discharge is of minor importance. The optical thickness is smaller for the wings of the line. The intensity for some frequencies exceeds that at the line centre. Beyond a given frequency offset the contribution of the quasi-static broadening becomes significant. As a result the spectral line shows a well pronounced asymmetry between the blue and red wings.

The spectral flux calculated both by means of the Monte Carlo simulation and the ray-tracing method is plotted in Fig. 3. The Monte Carlo simulation is performed with $N=3x10^5$ test histories. The error of this technique, being statistical, decreases as $1/N^{1/2}$. In order to compare the two results the latter have been re-normalized to the same curve area. The self-reversal is not so pronounced anymore since the outer region of the discharge contributes mainly at frequencies near the line centre, with spectral intensities showing no self-reversal (Fig. 2). The results obtained by the two approaches show a fair agreement with each other.

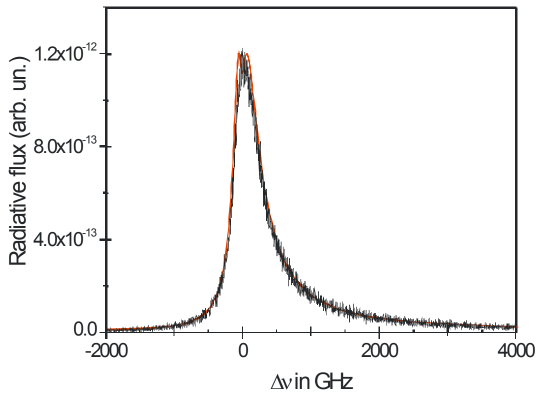

Fig. 3 Radiative flux computed by Monte Carlo simulations and the ray-tracing method.

The computational costs for both approaches depend on the optical thickness and the required precision. For the case study considered here (a problem with a cylindrical symmetry) the Monte Carlo simulation is more time consuming compared to the ray-tracing method. For complex geometries the Monte Carlo simulation is probably preferable.

5. Acknowledgments

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6. References

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